2011 Marywood Mathematics Contest

Level I

Sponsored by

SEMI-GROUP

The Student Mathematics Club of

Marywood University

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Directions:

1. This exam consists of 40 questions on 6 pages. Please check to make sure that you have all the pages.

2. No calculator or any other electronic device is allowed on this exam.

3. Allot your time accordingly. This is a 60-minute test. Do not spend too much time on any one problem. If a question seems to be too difficult, make your best possible guess. Your score will be the number of correct responses.

4. On the scantron form provided for you, darken in the space corresponding to the correct answer. Please mark all answers carefully and erase completely when changing an answer. Mark only one answer for each question. Only those answers on the answer sheet will be counted.

5. There is a sheet of blank paper on the last page which you can tear off and use as scratch paper. You may also use the back of the pages.

6. NOTE: In order to ensure uniformity, proctors are NOT allowed to answer any questions pertaining to specific problem content.

    Please do NOT open the test until you are told to do so.
1. Evaluate \( \left( 16 + \frac{-2^3 \times 10}{5} \right) \cdot \left( 3^7 - 2011^6 + 72 \right) \).
   A. 0  B. 1  C. 2  D. 3  E. None of these.

2. Solve the equation \( 7x + 3(4x - 5) = 0 \) for \( x \).
   A. \( \frac{1}{5} \)  B. \( \frac{5}{19} \)  C. \( \frac{15}{19} \)  D. \( \frac{5}{11} \)  E. None of these.

3. In which list are the three numbers arranged from smallest to largest?
   A. \( 2.32, \frac{7}{3}, \frac{12}{5} \)  B. \( \frac{7}{3}, \frac{12}{5}, 2.32 \)  C. \( \frac{12}{5}, 2.32, \frac{7}{3} \)  D. \( \frac{7}{3}, 2.32, \frac{12}{5} \)  E. None of these.

4. Which is the SMALLEST of these five numbers?
   A. \( \frac{2}{3} + \frac{1}{6} \)  B. \( \frac{2}{3} - \frac{1}{6} \)  C. \( \frac{2}{3} \times \frac{1}{6} \)  D. \( \frac{2}{3} \div \frac{1}{6} \)  E. \( \frac{1}{6} \div \frac{2}{3} \)

5. Find a number which is one third of the way from \( \frac{5}{6} \) to \( \frac{7}{2} \) on the number line.
   A. \( \frac{13}{9} \)  B. \( \frac{7}{3} \)  C. \( \frac{13}{3} \)  D. \( \frac{31}{18} \)  E. None of these.

6. If \( x \) and \( y \) are real numbers then \( \left( 32x^{16}y^{15} \right)^{\frac{1}{3}} = \)
   A. \( 2x^{16/5}y^{3} \)  B. \( 4x^{16/5}y^{3} \)  C. \( 8x^{16/5}y^{3} \)  D. \( 16x^{16/5}y^{3} \)  E. None of these.

7. Which linear equation has \( x \)- and \( y \)-intercepts at \((5, 0)\) and \((0, -12)\)?
   A. \( 5x - 12y + 144 = 0 \)  B. \( 12x - 5y + 144 = 0 \)  C. \( 5x + 12y + 144 = 0 \)
   D. \( 12x + 5y + 144 = 0 \)  E. None of these.

8. How many real solutions does \( x^2 - 5x + 4 = 0 \) have?
   A. 0  B. 1  C. 2  D. 3  E. None of these.

9. The roots of \( x(x^2 + 8x + 16)(4 - x) \) are
   A. 0  B. 0, 4  C. 0, -4  D. 0, 4, -4  E. None of these.
10. The sum of all positive integer factors of 48 is
   A. 75       B. 105       C. 108       D. 112       E. None of these.

11. A 27-year old mother has a 5-year old daughter. In how many years will the mother be three times as old as her daughter?
   A. 3       B. 4       C. 5       D. 6       E. None of these.

12. Suppose that the operation $\diamond$ is defined as $a \diamond b = \frac{a \cdot b + b}{2}$. What is the value of $3 \diamond (2 \diamond 6)$?
   A. 12       B. 14       C. 15       D. 18       E. None of these.

13. What is the value of $2^{22}$?
   A. 512       B. 256       C. 128       D. 64       E. None of these.

14. A woodchuck could chuck as much wood as a woodchuck could chuck, if a woodchuck could chuck wood. Assume that a woodchuck could chuck 3 cords of wood in 90 minutes. How many cords of wood could 4 woodchucks chuck in 2 hours?
   A. 12       B. 14       C. 16       D. 18       E. None of these.

15. Gillian has a collection of 50 songs that are each 3 minutes in length and 50 songs that are each 5 minutes in length. What is the maximum number of songs from her collection that she can play in 5 hours?
   A. 70       B. 80       C. 90       D. 100       E. None of these.

16. Barry’s daily grades for one grading period are shown below:

   94, 88, 87, 92, 78, 88, 93, 100, 92, 92, 90, 92, 85.

   What is the mode of his grades?
   A. 93       B. 92       C. 91       D. 90       E. None of these.
17. The minimum value of the parabola \( y = x^2 - 6x + 8 \) occurs at \( x = \)

A. 2  B. 3  C. 4  D. 5  E. None of these.

18. If \( x - y = 3 \) and \( x^2 - y^2 = 9 \), then \( xy = \)

A. 0  B. 1  C. 2  D. 3  E. None of these.

19. The prime factorization of 2000 can be written \( 2000 = 2^x \cdot 5^y \). The sum \( x + y \) equals

A. 6  B. 7  C. 8  D. 9  E. None of these.

20. If \( x + y = -23 \) and \( x - y = 7 \) then \( 2x - y \) equals

A. -31  B. -1  C. 1  D. 8  E. None of these.

21. To build a set of book shelves, a carpenter needs 30 identical boards, each 2 feet 7 inches long. For this type of wood, the lumber yard only sells boards in 12-foot lengths. How many 12-foot boards must the carpenter purchase?

A. 6  B. 7  C. 8  D. 9  E. None of these.

22. A math teacher buys three sport coats and six ties. He teaches Monday through Friday. Starting on a Monday, he wears a different combination of new sport coat and new tie each day that he teaches. On which day of the week does he wear his last new combination?

A. Monday  B. Tuesday  C. Wednesday  D. Thursday  E. None of these.

23. A Leap Year has 366 days and the year 2012 will be a Leap Year. Randy’s 28th birthday on April 1, 2011 will be on a Friday. Randy’s 29th birthday will be on a

A. Sunday  B. Monday  C. Tuesday  D. Wednesday  E. None of these.

24. Two standard six-sided dice are rolled. The probability that the sum of the two dice is 9 or larger is

A. \( \frac{5}{18} \)  B. \( \frac{1}{6} \)  C. \( \frac{1}{12} \)  D. \( \frac{1}{36} \)  E. None of these.
25. A square piece of paper is folded in half twice: from top to bottom, then from top to bottom again. If the perimeter of the final rectangle is 10 cm, what was the perimeter of the original square?

A. 14  B. 15  C. 16  D. 20  E. None of these.

26. Randomly select two different numbers from this set: \{-5, -2, 4, 8\}. The probability that the product of the two selected number is positive is

A. \( \frac{1}{6} \)  B. \( \frac{1}{3} \)  C. \( \frac{1}{2} \)  D. \( \frac{2}{3} \)  E. None of these.

27. What is the last digit of \( 2011^{2011} \)?

A. 0  B. 1  C. 2  D. 3  E. None of these.

28. If \( i^2 = -1 \), then \( i^{2011} = \)

A. 1  B. -1  C. \( i \)  D. \( -i \)  E. None of these.

29. If each edge of a cube is increased by 30%, by what percentage does the surface area of the cube increase?

A. 10%  B. 20%  C. 30%  D. 40%  E. None of these.

30. A farm consists of a right triangle and the three squares on the sides of the right triangle. The length of the three sides of the right triangle are \( a \), \( b \), and \( c \). The farmer decides to keep the triangular piece of land for himself. He shares the rest of the farm equally, in terms of area, between his two children.

What area of land does each child receive?

A. \( \left( \frac{a}{2} \right)^2 + \left( \frac{b}{2} \right)^2 + \left( \frac{c}{2} \right)^2 \)
B. \( \left( \frac{a + b + c}{2} \right)^2 \)
C. \( \frac{(a + b + c)^2}{2} \)
D. \( c^2 \)
E. None of these.
31. In triangle ABC, AC = 51 and BC = 50. Point D on AB divides it into segments of length AD = 1 and DB = 3. Which value best approximates the length of DC?

A. 50     B. 50.25     C. 50.5     D. 50.75     E. 60

32. In a 6-horse race with no ties, Family finished fifth, 14 meters behind Doggie and 20 meters behind Boya. Alpha finished 8 meters ahead of Eppa and 10 meters behind Captain. Which horse finished fourth?

A. Alpha     B. Boya     C. Captain     D. Doggie     E. None of these.

33. A cube measuring 1 cm on a side is sliced in half by a plane through vertices A, B, C, and D. Two congruent prisms are formed. The surface area, in square centimeters, of each prism is

A. \( \frac{1}{2} \)     B. 3     C. 4     D. 5     E. None of these.

34. The four vertices of a square in a rectangular coordinate system are (9, 6), (5, 16), (a, b), and (c, d). The vertices (9, 6) and (5, 16) lie on one diagonal of the square. Then the sum \( a + b + c + d \) equals

A. 15     B. 20     C. 25     D. 30     E. None of these.

35. In a three-digit whole number N, the hundreds digit is the same as the units digit. When N is divided by the sum of its digits, the quotient is 28. The tens digit of N is

A. 2     B. 3     C. 4     D. 5     E. None of these.

36. The natural numbers are written in sequence, without spaces:

1234567891011121314151617....

and so on. What is the 100th digit in this sequence?

A. 1     B. 3     C. 4     D. 6     E. None of these.
37. You have one sheet of very thin paper. One stack of 200 sheets is only one centimeter tall. If you could fold the paper in half 50 times, how thick, in kilometers, would the folded paper be? (Select the closest answer.)

A. 1  B. 5  C. 500  D. 50,000  E. 50,000,000

38. The first four figures below consist of 1, 5, 13, and 25 squares respectively. If the pattern in the figures continues, how many squares will there be in figure 100?

A. 39801  B. 20201  C. 19801  D. 10401  E. None of these.

39. In rectangle $ABCD$, $AD = 1$, $P$ is on $AB$, and the line segments $DB$ and $DP$ trisect $\angle ADC$. What is the perimeter of $\triangle BDP$?

A. $2 + 2\sqrt{2}$  B. $2 + \frac{5\sqrt{3}}{3}$  C. $2 + \frac{4\sqrt{3}}{3}$  D. $3 + \frac{\sqrt{3}}{3}$  E. None of these.

40. In the figure shown, if the area of the letter “L” equals the area of the triangle, what is the length $x$ of the ends of the $L$?

A. $\frac{6 - 2\sqrt{6}}{3}$  B. $\frac{6 - 3\sqrt{6}}{3}$
C. $\frac{6 + 2\sqrt{6}}{3}$  D. $\frac{6 + 3\sqrt{6}}{3}$  E. None of these.